

1. $\int x^2(x^3 - 1)^4 dx$

$$\begin{aligned} u &= x^3 - 1 & \int x^2 u^4 \cdot \frac{du}{3x^2} \\ \frac{du}{dx} &= 3x^2 & \int \frac{u^4}{3} du \\ du &= 3x^2 dx & \frac{u^5}{15} + C \\ dx &= \frac{du}{3x^2} & \frac{(x^3 - 1)^5}{15} + C \end{aligned}$$

3. $\int \frac{x^2}{(1+x^3)^2} dx$

$$\begin{aligned} u &= 1+x^3 & \int \frac{x^2}{u^2} \cdot \frac{du}{3x^2} \\ \frac{du}{dx} &= 3x^2 & \int \frac{1}{3} u^{-2} du \\ dx &= \frac{du}{3x^2} & -\frac{1}{3} u^{-1} + C \\ & & \frac{-1}{3} (1+x^3)^{-1} + C \end{aligned}$$

5. $\int \frac{1}{\theta^2} \cos \frac{1}{\theta} d\theta$

$$\begin{aligned} u &= \theta^{-1} & \int \frac{1}{\theta^2} \cos u \cdot -\theta^{-2} du \\ \frac{du}{d\theta} &= -\theta^{-2} & \int -\cos u du \\ \frac{du}{d\theta} &= -\frac{1}{\theta^2} & -\sin u + C \\ du \cdot \theta^2 d\theta & & -\sin \frac{1}{\theta} + C \end{aligned}$$

7. $\int \frac{3x}{\sqrt[3]{10-x^2}} dx$

$$\begin{aligned} u &= 10-x^2 & \int \frac{3x}{u^{1/3}} \cdot \frac{du}{-2x} \\ \frac{du}{dx} &= -2x & \int -\frac{3}{2} u^{-1/3} du \\ dx &= \frac{du}{-2x} & -\frac{3}{2} u^{2/3} \cdot \frac{3}{2} + C \\ & & -\frac{9}{4} (10-x^2)^{2/3} + C \end{aligned}$$

2. $\int 5x^3 \sqrt{1-x^2} dx$

$$\begin{aligned} u &= 1-x^2 & \int 5x u^{1/2} \cdot \frac{du}{-2x} \\ \frac{du}{dx} &= -2x & \int -\frac{5}{2} u^{1/2} du \\ dx &= \frac{du}{-2x} & -\frac{5}{2} u^{4/3} \cdot \frac{3}{4} \\ & & -\frac{15}{8} (1-x^2)^{4/3} + C \end{aligned}$$

4. $\int \sin 2x dx$

$$\begin{aligned} u &= 2x & \int \sin u \cdot \frac{du}{2} \\ \frac{du}{dx} &= 2 & \int \frac{1}{2} \sin u du \\ dx &= \frac{du}{2} & -\frac{1}{2} \cos u + C \\ & & -\frac{1}{2} \cos 2x + C \end{aligned}$$

6. $\int \sin 2x \cos 2x dx$

$$\begin{aligned} u &= \sin 2x & \int u \cos 2x \cdot \frac{du}{2 \cos 2x} \\ \frac{du}{dx} &= \cos 2x \cdot 2 & \int \frac{1}{2} u du \\ dx &= \frac{du}{2 \cos 2x} & \frac{1}{4} u^2 + C \\ & & \frac{\sin^2 2x}{4} + C \end{aligned}$$

8. $\int x^3 \sqrt{5x^4 + 20} dx$

$$\begin{aligned} u &= 5x^4 + 20 & \int x^3 u^{1/2} \cdot \frac{du}{20x^3} \\ \frac{du}{dx} &= 20x^3 & \int \frac{1}{20} u^{1/2} du \\ dx &= \frac{du}{20x^3} & \frac{1}{20} u^{3/2} \cdot \frac{2}{3} + C \\ & & \frac{1}{30} (5x^4 + 20)^{3/2} + C \end{aligned}$$

$$9. \int \frac{1}{(x-1)^2} dx$$

$$\begin{aligned} u &= x-1 & \int \frac{1}{u^2} du \\ \frac{du}{dx} &= 1 & \int u^{-2} du \\ du &= dx & -u^{-1} + C \\ & & -\frac{1}{(x-1)} + C \end{aligned}$$

$$11. \int \frac{1}{\sqrt{x}} \sin \sqrt{x} dx$$

$$\begin{aligned} u &= x^{1/2} & \int \frac{1}{\sqrt{x}} \sin u \cdot 2\sqrt{x} du \\ \frac{du}{dx} &= \frac{1}{2} x^{-1/2} & \int 2 \sin u du \\ \frac{du}{dx} &= \frac{1}{2\sqrt{x}} & -2 \cos \sqrt{x} + C \\ dx &= 2\sqrt{x} du & \end{aligned}$$

$$13. \int \frac{\cos\left(\frac{3}{x}\right)}{x^2} dx$$

$$\begin{aligned} u &= 3x^{-1} & \int \frac{\cos u}{x^2} \cdot \frac{x^2 du}{-3} \\ \frac{du}{dx} &= -3x^{-2} & \int -\frac{1}{3} \cos u du \\ \frac{du}{dx} &= -\frac{3}{x^2} & -\frac{1}{3} \sin\left(\frac{3}{x}\right) + C \\ x^2 du &= -3dx & \\ dx &= \frac{x^2 du}{-3} & \end{aligned}$$

$$15. \int \sin(\sin x) \cos x dx$$

$$\begin{aligned} u &= \sin x & \int \sin u \cos x \cdot \frac{du}{\cos x} \\ \frac{du}{dx} &= \cos x & \int \sin u du \\ \frac{du}{dx} &= \frac{du}{\cos x} & -\cos(\sin x) + C \end{aligned}$$

$$10. \int (x^2+1)(x^3+3x)^{-5} dx$$

$$\begin{aligned} u &= x^3+3x & \int (x^2+1) u^{-5} \cdot \frac{du}{3(x^2+1)} \\ \frac{du}{dx} &= 3x^2+3 & \int \frac{1}{3} u^{-5} du \\ \frac{du}{dx} &= \frac{du}{3(x^2+1)} & -\frac{1}{12} u^{-4} + C \\ & & -\frac{1}{12} (x^3+3x)^{-4} + C \end{aligned}$$

$$12. \int x^2 \sec^2 x^3 dx$$

$$\begin{aligned} u &= x^3 & \int x^2 \sec^2 u \cdot \frac{du}{3x^2} \\ \frac{du}{dx} &= 3x^2 & \int \frac{1}{3} \sec^2 u du \\ \frac{du}{dx} &= \frac{du}{3x^2} & \frac{1}{3} \tan x^3 + C \end{aligned}$$

$$14. \int \frac{\sin 2x}{(1-\cos 2x)^3} dx$$

$$\begin{aligned} u &= 1-\cos 2x & \int \frac{\sin u}{u^3} \cdot \frac{du}{2\sin 2x} \\ \frac{du}{dx} &= \sin 2x \cdot 2 & \int \frac{1}{2} u^{-3} du \\ \frac{du}{dx} &= \frac{du}{2\sin 2x} & -\frac{1}{4} u^{-2} + C \\ & & -\frac{1}{4} (1-\cos 2x)^{-2} + C \end{aligned}$$